Jrl Syst Sci & Complexity (2006) 19: 54–62

SOFT CONTROL ON COLLECTIVE BEHAVIOR OF A GROUP OF AUTONOMOUS AGENTS BY A SHILL AGENT*

Jing HAN · Ming LI · Lei GUO

Received: 26 September 2005

Abstract This paper asks a new question: how can we control the collective behavior of self-organized multi-agent systems? We try to answer the question by proposing a new notion called 'Soft Control', which keeps the local rule of the existing agents in the system. We show the feasibility of soft control by a case study. Consider the simple but typical distributed multi-agent model proposed by Vicsek et al. for flocking of birds: each agent moves with the same speed but with different headings which are updated using a local rule based on the average of its own heading and the headings of its neighbors. Most studies of this model are about the self-organized collective behavior, such as synchronization of headings. We want to intervene in the collective behavior (headings) of the group by soft control. A specified method is to add a special agent, called a 'Shill', which can be controlled by us but is treated as an ordinary agent by other agents. We construct a control law for the shill so that it can synchronize the whole group to an objective heading. This control law is proved to be effective analytically and numerically. Note that soft control is different from the approach of distributed control. It is a natural way to intervene in the distributed systems. It may bring out many interesting issues and challenges on the control of complex systems.

Key words Boid model, collective behavior, multi-agent system, shill agent, soft control.

1 Introduction

Collective behavior is the high level (macroscopic) property of a self-organized system which consists of a large number of (microscopic) individuals (agents). Examples are synchronization, aggregation, phase transition, pattern formation, swarm intelligence, fashion, etc. People found this kind of phenomena in many systems, such as flocking of birds, schools of fish, cooperation in ant colonies, panic of crowds[1], norms in economic systems[2], etc. Without any question, collective behavior is one of the fundamental and difficult topics of the study of complex systems. We classify the research on collective behavior into three categories.

(I) Given the local rules of agents, what is the collective behavior of the overall system?

Jing HAN
Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100080, *China*. Email: hanjing@amss.ac.cn.
Ming LI
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, *China*.
Lei GUO
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100080, *China*.
*This work was supported by the National Natural Science Foundation of China (No. 20336040, No. 60574068, and No. 60221301).

Many people have been working on this category which is about the mechanism of how collective behavior emerges from multi-agent systems. “More is different”[3]. The physicists have applied theory of statistical physics to explored some simple models, from the Ideal Gas model, Spin Glasses, to the panic model and network dynamics.

(II) Given the desired collective behavior, what are the local rules for agents?

Some people work on this category. One typical example is Swarm Intelligence. Since the high level function of the overall system can be more than the sum of all individuals, how do we construct robust intelligence by a large number of locally interacting simple agents? An ant is simple and often moves randomly. But a colony of ants can efficiently find the shortest path from their nest to a food source. This natural phenomenon inspired the Ant Colony Optimization Algorithm[4].

(III) Given the local rules of agents, how we control the collective behavior?

In some real applications, it is very difficult or even impossible to change the local rules of agents, such as the behavioral rules of people in panic and the flying strategies of birds. Yet we need to control the system to avoid danger or improve efficiency. Then what is the feasible way to intervene in the collective behavior? There should be a way especially for the multi-agent system that utilizes the property of collective behavior. This question is not well noticed in the control literature. In this paper, we propose a new control notion, called ‘soft control’[1], which keeps basic local rule of the existing agents in the system. There are no global parameters can be adjusted to achieve the control purpose, such as adjusting the temperature to change water from liquid phase to gas/solid phase or broadcasting orders directly to all agents by a controller. This feature makes a difference between soft control and the traditional control.

These three categories are tightly related. The first category (I) is about how a distributed system behaves and what emerges from the system. Both of (II) and (III), however, require the system to behave at the high level as what we expected, i.e., we know what collective behavior should be emerged from the system. But (II) focuses on how to design a distributed system by constructing(changing) the local rules for interaction among agents, and (III) focuses on how to intervene in the existing system by a nondestructive way.

Since collective behavior is a kind of macroscopic feature of a multi-agent system, usually adding one or a few more agents will not affect it. But if those agents are special ones which can be controlled, even though only a small number (in some cases, only one), it can dramatically change the collective behavior as will be shown in Section 3. So in this paper, a way to control the system without changing the local rules of the existing agents is to add one (or a few) special agent which can be controlled. Therefore its behavior does not necessarily obey the local rules as the ordinary agents do. However, the existing agents still treat the special agent as an ordinary agent, so the special agent only affects the local area with limited power. The special agent is the only controlled part of the system and it indeed changes the collective behavior of the system by ‘cheating’ and ‘seducing’ its neighboring ordinary agents. So we do not call the special agent a leader. Instead, we call it a ‘Shill’.

Soft control is different from distributed control. Distributed control recently gets more and more attention because many real-world applications are distributed systems, such as power networks and traffic systems. The system consists of many interacting subsystems that all have their own controllers with local feedback. The study of the relationship about performance between the overall system and the subsystems falls into category (I). On the other hand, designing a practical distributed system for resource sharing and cooperation is a demanding and complicated task. This research is of the category (II). So we can see that both soft control and

1 The idea of soft control in this paper was inspired by discussions at the “Systems, Control and the Complexity Science” Xiangshan Science Conference (May, 2004, Beijing) and the preliminary result was presented at the 2nd Chinese-Swedish Conference on Control[5].

distributed control concern with the macroscopic collective behavior (such as synchronization) of the self-organized multi-agent system with local rules. But in distributed control, every agent is treated as a control system and has its control law (which is the set of local rules); while in soft control framework, all these agents are treated as one system and the control law is for the shill. So soft control can be regarded as a way of intervention in the distributed systems. With the growing literature on complex systems, the challenge of controlling complex systems is likely to become a key problem for control scientists.

This paper is going to explore the idea of soft control by a case study, where we will show how a shill can be used to control the headings of a group of mobile agents demonstrated based on the modified Boid model[6] proposed by Vicsek et al.[7]. The current control theory cannot be applied directly to this problem because our soft control is a nondestructive intervention in a locally interacting autonomous multi-agent system, which is not allowed to change the local rules of the existing agents. The formation control[8] mainly focuses on how to design the local rules of a team of robots to maintain a geometric configuration during movement, which is a very special kind of collective behavior, and its approach belongs to distributed control. The pinning control[9] is especially for stabilizing dynamical networks (with special topology, such as random networks, small-world and scale-free networks) by imposing controllers on a small fraction of selected nodes in the network. No one appear to have directly studied the above mentioned soft control problem in the literature. Recently, Jadbabie et al.[10] investigated a multi-agent model which is slightly different from the one proposed by Vicsek et al., they showed that under some a priori connectivity conditions, the group will eventually move in the same direction even without centralized control. This is a problem of the category (I) of collective behavior study. An idea in [10] worthy of mention is the Leader Following model, which has a leader agent with fixed heading. But it did not tell how to implement synchronization by a leader. The so-called 'virtual leader'[11] is not treated as an agent but part of the potential, which requires to introduce new rules for the ordinary agents to recognize it.

The rest of this paper is organized as follows: The modified BOID model proposed by Vicsek et al. and the notion of soft control are set up in Section 2. Section 3 is a case study for soft control. We will solve a simple but non-trivial problem by constructing an effective control law for the shill. Both theoretical analysis and computer simulation show that one shill is possible to turn the direction of the whole group. Some concluding remarks are given in Section 4.

2 Soft Control of the Modified Boid Model

Eighteen years ago, Renolds proposed a simultaneous discrete-time multi-agent model, which is called Boid[6], to catch the natural phenomenon of flocking of birds and fish and make computer animation. In this model, each bird decides its flying direction only by looking at its current neighboring birds and using three rules (Alignment, Separation and Cohesion) based on the status of its neighbors. These rules are local and simple but the overall system exhibits the flocking behavior. The Boid model became very popular in complex systems studies. Unfortunately, this simple model is not simple at all for theoretical analysis.

In 1995, to investigate the emergence of self-ordered motion, Vicsek et al.[7] proposed a model which is actually a modified version of Boid, by only keeping the Alignment rule– steer towards the average heading of neighbors, which still can catch the flocking behavior. There are n agents $(x_i(\cdot), \theta_i(\cdot))$ for n birds, labelled from 1 through n, all moving simultaneously in the two-dimensional space. The velocity of agent i at time t is defined by

$$\boldsymbol{v}_i(t) = (v\cos(\theta_i(t)), v\sin(\theta_i(t))), \tag{1}$$

which is constructed to have an absolute value v and a heading given by the angle $\theta_i(t) \in [0, 2\pi)$.

The position of agent i at time t is denoted as $\boldsymbol{x}_i(t)$. The neighborhood of agent i at time t is defined as $N_i(t) = \{j \mid \|\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)\| \le r, j = 1, 2, \cdots, n\}$, where r is the radius of the neighborhood circle. For any agent i, its heading and position are updated by (2) and (3) below if ignoring noise:

$$\theta_i(t+1) = \langle \theta_i(t) \rangle_r, \tag{2}$$

$$\boldsymbol{x}_i(t+1) = \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t+1), \tag{3}$$

where $\boldsymbol{v}_i(t+1)$ is defined by (1), and $\langle \theta_i(t) \rangle_r$ is the angle of the sum of the velocity vectors of neighbors of agent i: $\sum\limits_{j \in N_i(t)} \boldsymbol{v}_j(t)$. In other words, $\langle \theta_i(t) \rangle_r$ can be obtained by

$$\arctan \left(\frac{\sum\limits_{j \in N_i(t)} \sin(\theta_j(t))}{\sum\limits_{j \in N_i(t)} \cos(\theta_j(t))} \right) \tag{4}$$

with some necessary regulations reflecting angles of $[0, 2\pi)$.

Some recent results of Jadbabaie et al.[10] have shown that under some connectivity conditions, the group will eventually move in the same direction. However, the model of [10] calculates $\langle \theta_i(t) \rangle_r$ by simply taking the average of angles of all neighbors. This brings convenience for mathematical analysis, but the system will sometimes exhibit counter-intuitive phenomena as they claimed in their paper[2]. So this paper keeps the updating rule of the model proposed by Vicsek et al., even though the analysis result about synchronization of the model in [10] cannot be utilized[3].

One can think of many related questions: what is the way to help the group to synchronize if the group will not synchronize by self-organization? What if the group self-organize to a synchronized direction α which is not what we want? What if we want the group to fly to a desired destination? And so on.

In this paper, we consider the case of adding one shill to control the collective behavior with the shill denoted as $(\boldsymbol{x}_0, \theta_0)$. The ordinary agent ($i = 1, 2, \cdots, n$) still keep the local rule as formula (2)–(3) to update its heading and position. The only difference is that the neighborhood $N_i(t)$ for agent i at time t will consider the shill: $N_i(t) = \{j \mid \|\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)\| \le r, j = 0, 1, 2, \cdots, n\}$. And the shill does not exactly obey the local rules as the ordinary agents do, its position and heading is decided by the control law u:

$$(\boldsymbol{x}_0(t), \theta_0(t)) = u(\boldsymbol{x}_1(t), \boldsymbol{x}_2(t), \cdots, \boldsymbol{x}_n(t), \theta_1(t), \theta_2(t), \cdots, \theta_n(t), t).$$

Note that $(\boldsymbol{x}_0, \theta_0)$ may be subject to some constraints. For example, the constraint of $\boldsymbol{x}_0(t+1) = \boldsymbol{x}_0(t) + \boldsymbol{v}_0(t+1)$ will make the shill behave like an ordinary agent except the way it decides its own heading. Without any constraint, the shill can fly anywhere with a 'deceptive' heading.

The control law u will be different for different control purposes/tasks, such as to synchronize headings of the group (n agents), to keep the group connected or to dissolve a group, to turn headings of the group (minimal circling), to lead the group to a destination (in a shortest time), to avoid hitting an object, etc.

There are many problems to be considered in terms of control: under what conditions we can use the shill to control the state of the multi-agent system (the controllability problem);

2 This is because the zero angle is special than other angles in $(0, 2\pi)$. The angles do not symmetrically behave.

3 The result about synchronization for the model of [10] is not true for the model of Vicsek et al.. Here is a counterexample: when $v = 0$, six agents are regularly put on the edge of a circle with radius of r, which form an ordinary hexagon. So each agent has three neighbors (because itself is also counted). The headings are $0, \pi/3, 2\pi/3, \pi, 4\pi/3, 5\pi/3$. This case will synchronize for the model in [10]. But it will not for the model we consider here, i.e., the Vicsek et al. model without noise.

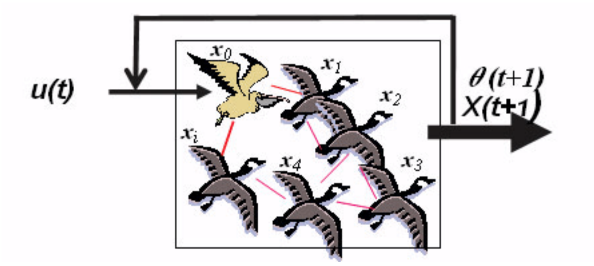


Figure 1 The soft control for the bird flocking model, by adding a shill. Two birds are linked if they are neighbors of each other at time t

which kind of state information can be used for control (the observation problem), e.g., the shill can only see some nearest agents but not all other agents; how can one design the control law when the local rule and other information is not clear (the learning and adaptation problem), etc. We do not intend to answer all these questions in this paper. As a beginning, we will just show how it works for a specific case, which is to synchronize and turn the headings of a group.

3 A Case Study

Flocking of birds in airports is dangerous. How we drive them away? In this case, obviously we can not use centralized control, such as broadcasting orders to the birds to change to the desired flying direction. The current solution is to use cannon to shoot them away. But can we drive them away by soft control? If the modified Boid model proposed by Vicsek et al. works for real birds flocking, and if we can make a controllable powerful robot bird, can we use this robot bird as the shill and guide flight of the birds? In this section, we will study a simple but non-trivial related problem[4].

The problem we consider here is: for a group of n agents with initial heading of $\theta_i(0) \in [0, \pi)$, $1 \le i \le n$, what is the control law for the shill, so that all agents will move to the direction of π eventually?

Suppose the local rule about the ordinary agents is known. Suppose also that the position $\boldsymbol{x}_0(t)$ and heading $\theta_0(t)$ of the shill can be controlled at any time step t. Suppose further that the state information (headings and positions) of all ordinary agents are observable at any time step. Now we propose an effective control law u_β which is defined as (see also Fig. 2):

$$x_0(t) = x_{s(t)}(t), \tag{5}$$

$$\theta_0(t) = \begin{cases} \theta_{s(t)}(t) + \beta, & \text{if} \quad \theta_{s(t)} \le \pi - \beta; \\ \pi, & \text{if} \quad \theta_{s(t)} > \pi - \beta, \end{cases} \tag{6}$$

where $\beta \in (0, \pi)$ is a constant, and $s(t)$ is defined as the 'worst' agent (or one of the 'worst' agents): $s(t) = \underset{1 \le i \le n}{\operatorname{argmin}} \{\theta_i(t)\}$;

The intuition behinds u_β is to put the shill to the position of the 'worst' agent $s(t)$ at each time step t and try to 'pull' it to the desired direction π. This greatly reduces the search space of positions and simplifies the control strategy. Note that we can not use $\theta_0(t) = \pi$ for all the

4 In flocking behavior of real animals, more factors such as different sensory systems should be considered. The Boid model is an abstract model which successfully simulates the flocking behavior. But no one has proved it to be true for real birds. So in this paper we just want to use a simple model to demonstrate how soft control works for self-organized multi-agent systems in general, but not to solve the specified airport birds problem.

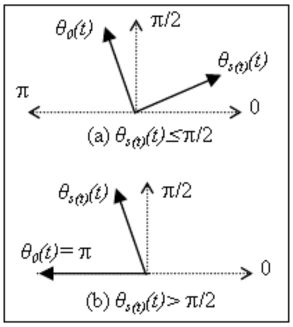


Figure 2 The control law u_β when $\beta = \pi/2$

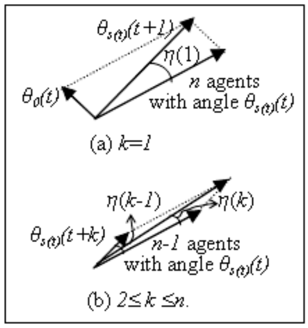


Figure 3 The worst cases where $\eta(k)$ has the smallest value

time because: (**A**) in the case where $\theta_{s(t)}(t) = 0$ and the all the neighboring ordinary agents of the shill have headings of zero degree, the shill with $\theta_0(t) = \pi$ can not change $\theta_{s(t)}(t)$ according to the update rule (2); (**B**) to avoid the ill-case where both of the numerator and denominator in (4) are zero[5]. But in the final stages we do need $\theta_0 = \pi$ to help the system to synchronize.

In the sequel, we use $\Delta(t) = \pi - \theta_{s(t)}(t)$ to denote the distance between the angel of the 'worst' agent and the objective angel π at time t. We now have the following main technical result of the paper:

Theorem 1 *For any $n \geq 2$, $r \geq 0$, $v \geq 0$, and any initial headings and positions $\theta_i(0) \in [0, \pi)$, $x_i(0) \in R^2$, $(1 \leq i \leq n)$, the update rule* (2)–(3) *and the soft control law u_β defined by* (5)–(6) *will lead to the asymptotic synchronization of the group, i.e., $\lim\limits_{t\to\infty} \Delta(t) = 0$.*

Proof Because $\pi \geq \theta_0(t) > \theta_{s(t)}(t) \geq 0$, and $\pi > \theta_i(t) \geq \theta_{s(t)}(t)$ for $1 \leq i \leq n$, it is obvious by the update rule (2) that $\pi > \theta_i(t+1) \geq \theta_{s(t)}(t)$ for $1 \leq i \leq n$ which implies that $\theta_{s(t)}(t) \leq \theta_{s(t+1)}(t+1)$. Hence by definition of $\Delta(t)$, we have $\Delta(t) \geq \Delta(t+1) \geq 0$, so $\Delta(t)$ will converge to some limit $\mu \geq 0$. If $\mu > 0$, then for any $0 < \varepsilon < \mu$, we know that $\Delta(t) \geq \varepsilon > 0$ holds for all large t. Consequently, by letting $t \to \infty$ in the following Lemma 2, we have $\mu \leq \mu - \delta$ which contradicts with $\delta > 0$. So we can conclude $\mu = 0$. This completes the proof of Theorem 1.

The main task now is to prove the following lemma.

Lemma 2 *If for some $t > 0$ and $\varepsilon > 0$, $\Delta(t) \geq \varepsilon > 0$, then there exists a constant $\delta \in (0, \varepsilon)$ which does not depend on t, such that $\Delta(t+n) \leq \Delta(t) - \delta$.*

Proof At any time step t, the shill is affecting the agent $s(t)$ with heading $\theta_0(t)$. We now proceed to analyze the lower bound to the angle change $\theta_{s(t)}(t+k) - \theta_{s(t)}(t)$ for $1 \leq k \leq n$, which is denoted by $\eta(k)$. For $k = 1$, the shill is affecting the agent $s(t)$, so the worst case where $\eta(1)$ has the smallest value is that the agent $s(t)$ is surrounded by $n-1$ agents with heading $\theta_{s(t)}(t)$ (see Fig. 3-(a)). For $k \geq 2$, however, the shill may not affect the agent $s(t)$, so

5 In the ill-case, $\sum\limits_{j \in N_i(t)} \sin(\theta_j(t)) = 0$ implies $\theta_j(t) = 0$ or π for all $j \in N_i(t)$ because $\sin\alpha > 0$ for $\alpha \in (0, \pi)$. Consequently, by $\sum\limits_{j \in N_i(t)} \cos(\theta_j(t)) = 0$ we further conclude that half of the headings should be zero and the other half should be π. So there must be at least one agent $j \neq 0$ such that $\theta_j(t) = 0$ (since $\theta_0(t) > 0$ for $t > 0$ by the control law), which means $\theta_{s(t)}(t) = 0$, and so we must have $\theta_0(t) = \beta \in (0, \pi)$ if $0 \in N_i(t)$. However, this will contradict with the fact that $\theta_j(t) = 0$ or π for all $j \in N_i(t)$. On the other hand, if $0 \notin N_i(t)$, $\sum\limits_{j \in N_i(t)} \sin(\theta_j(t)) = 0$ only holds when $\theta_j(t) = 0$ for all $j \in N_i(t)$ since obviously $\theta_j(t) < \pi$ for all $j = 1, \cdots, n$ and $t \geq 0$, but in this case we have $\sum\limits_{j \in N_i(t)} \cos(\theta_j(t)) > 0$. So the ill-case will never happen in (4).

the worst case where $\eta(k)$ has the smallest value is that the agent $s(t)$ is surrounded by $n-1$ agents with heading $\theta_{s(t)}(t)$ again, but without the shill in its neighborhood (see Fig. 3-(b)). Now by the definition of $\eta(k)$ we have

$$\theta_{s(t)}(t+k) \geq \theta_{s(t)}(t) + \eta(k), \tag{7}$$

and a simple calculation will give

$$\eta(k) = \begin{cases} \min\left\{ \arctan\dfrac{\sin\beta}{n+\cos\beta},\ \arctan\dfrac{\sin\varepsilon}{n+\cos\varepsilon} \right\}, & k=1; \\ \arctan\dfrac{\sin(\eta(k-1))}{n-1+\cos(\eta(k-1))}, & 2 \leq k \leq n. \end{cases} \tag{8}$$

First, note that $\Delta(t) \geq \varepsilon > 0$ and the function $\psi(\alpha) = \arctan(\sin\alpha/(m+\cos\alpha)), (m>0)$ is monotonically increasing when $\alpha \in [0, \arccos(-1/m)]$ and monotonically decreasing when $\alpha \in [\arccos(-1/m), \pi]$.

Second, to explain the definition of $\eta(1)$ in (8) we need to show that

$$0 < \eta(1) \leq \arctan\frac{\sin(\theta_0(t) - \theta_{s(t)}(t))}{n + \cos(\theta_0(t) - \theta_{s(t)}(t))}. \tag{9}$$

In fact, by the control law (6) and the expression of $\eta(1)$ in (8), the above inequality (9) is trivial in the case where $\theta_{s(t)} \leq \pi - \beta$. So we only need to consider the case where $\theta_{s(t)} > \pi - \beta$. In this case the control law is $\theta_0(t) = \pi$. We consider two subcases as follows:

(i) $\theta_0(t) - \theta_{s(t)}(t) \geq \arccos(-1/n)$. Since $\theta_0(t) - \theta_{s(t)}(t) \leq \pi - (\pi - \beta) = \beta$, by the monotonicity of the function $\psi(\cdot)$, we know that the claim (9) is true.

(ii) $\theta_0(t) - \theta_{s(t)}(t) < \arccos(-1/n)$. Since $\theta_0(t) - \theta_{s(t)}(t) = \Delta(t) \geq \varepsilon$, by the monotonicity of the function $\psi(\cdot)$ again, we know that the claim (9) is also true.

Hence, (9) holds in any case.

Now, note that $0 < \eta(1) < \pi/2$, and again by the monotonicity of the function $\psi(\cdot)$, we have $0 < \eta(n) < \eta(n-1) < \cdots < \eta(1) < \pi/2$.

Let us take $\delta = \eta(n)$ which is a finite positive value depending on n, β and ε only. Obviously, we have $\theta_{s(t)}(t+n) - \theta_{s(t)}(t) \geq \delta$.

Next, we consider the other agents: $i = 1, 2, \cdots, n, i \neq s(t)$.

Let $\Lambda(k)$ denote the set of agents whose angles are inside $(\theta_{s(t)}(t), \theta_{s(t)}(t)+\delta)$ at time step $t+k$, i.e.,

$$\Lambda(k) = \{ i \mid \theta_{s(t)}(t) < \theta_i(t+k) < \theta_{s(t)}(t) + \delta,\ i = 1, 2, \cdots, n\}.$$

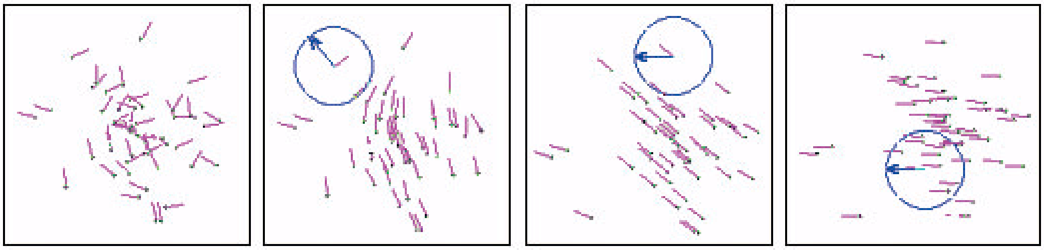
We now proceed to show that we must have $\Lambda(n) = \emptyset$.

First of all, as shown above, $s(t)$ does not belong to $\Lambda(k)$ for $1 \leq k \leq n$ because $\theta_{s(t)}(t+k) \geq \theta_{s(t)}(t) + \eta(k) \geq \theta_{s(t)}(t) + \delta$. So we have $|\Lambda(k)| \leq n-1$ for $1 \leq k \leq n$.

Therefore, at time step $t+1$, there are at most $n-1$ agents with angles which are less than $\theta_{s(t)}(t) + \delta$. If $\Lambda(1) \neq \emptyset$, $s(t+1)$ will be picked up from $\Lambda(1)$. The shill will change the heading of agent $s(t+1)$. Because $\theta_{s(t+1)}(t+1) \geq \theta_{s(t)}(t)$ and $\theta_{s(t)}(t+k) \geq \theta_{s(t)}(t) + \eta(k)$ for any t, we have for $k \geq 2$,

$$\theta_{s(t+1)}(t+k) \geq \theta_{s(t+1)}(t+1) + \eta(k-1) \geq \theta_{s(t+1)}(t+1) + \delta \geq \theta_{s(t)}(t) + \delta.$$

So $s(t+1)$ will not belong to $\Lambda(k)$ for any $k \in [2, n]$, we have $|\Lambda(k)| \leq n-2$ for $2 \leq k \leq n$. By repeating this argument, we get for $1 \leq d \leq k \leq n$, $\theta_{s(t+d)}(t+k) \geq \theta_{s(t)}(t) + \delta$ and



(a) $t = 0$ (b) $t = 1$ (c) $t = 43$ (d) $t = 178$

Figure 4 A simulation for a group of $n = 50$ and $\beta = \pi/2$. The ordinary agents are represented by the small black dots with the line point to the direction they are moving. The shill is the one with arrow showing its heading direction and the circle showing the current neighborhood radius of all agents. On the other hand, the circle of the shill indicates its effective area. The system starts from a random initialization (a). $\Delta \to 0$ in (d).

$|\Lambda(k)| \leq n - d$. So by taking $d = k = n$, we have $|\Lambda(n)| = 0$. This means that after at most n steps, Λ will be empty.

Consequently, we know that $\theta_i(t+n) \geq \theta_{s(t)}(t) + \delta$ for all i. Then we get $\theta_{s(t+n)}(t+n) \geq \theta_{s(t)}(t) + \delta \Rightarrow \Delta(t+n) \leq \Delta(t) - \delta$. This complete the proof of Lemma 2.

The snapshots of the relating computer simulation are showed in Figure 4. The video of demo and the simulation program can be downloaded from the project website[12].

Although our case study is a simple starting point, it shows that it is possible to change the collective behavior (headings) of a group by soft control. Based on this work, we are going to explore more complicated control problems, such as what if the energy of the shill is concerned (the energy of the shill is limited so that it can not 'jump' as in (5)), what if the shill only can see locally, and what if the shill also needs to keep the group connected while turning the group, etc.

5 Concluding Remarks

The collective behavior of complex systems is the macroscopic feature that we concerned and studied in this paper. How we control or intervene in a multi-agent system without changing the local rule of the existing agents is an important issue but has not been well recognized yet. The idea of soft control that we introduced and explored here appear to be an initial attempt to tackle this issue. We believe that our general soft control framework reflects a large class of control problems in complex systems.

In this paper, we have restricted ourselves on the study of the modified Boid model which does catch certain key properties of many real world complex systems (see, e.g., [6,7,10]). Without changing the local rule of the existing agents, we have added a shill in the distributed system and designed the control law for the shill carefully for the control purpose. Although the shill does not behave as the ordinary agents do, it is not recognized by the ordinary agents and is therefore still treated as an ordinary agent by them. This soft control idea works well as we have shown in the case study. We would like to emphasis that the idea of adding a shill is just one of the methods of soft control, and for different systems, they would be certainly different.

There are many potential applications of soft control. Currently a project which is called Claytronics[13], a new form of programmable matter, is to study how to form interesting dynamic shapes and configurations by simple and local interactions of a billion micro-scale units. This research is in the category (II) of collective behavior. Using soft control to help to form shapes may release us from designing subtle local rules. Another possible case is to use soft control in the language diffusion model to guide evolution of language, and it might be able to save dying language[14]. A recent study in economics[2] shows that a small proportion of special agents who have preferences can dramatically alter the expected waiting time between norm transitions in a population of agents who repeatedly play the Nash demand game. If those special agents have feedbacks, how does the soft control help changing the transition of norms more efficiently? How we use soft control to avoid and handle panic of crowd[1]? $\cdots$. In conclusion, we need a theory of soft control to efficiently intervene in many complex systems.

References

[1] D. Helbing, I. Farkas, and T. Vicsek, Simulating dynamical features of escape panic, *Nature*, 2000, **407**: 487–490.

[2] R. L. Axtell and S. Chakravarty, Radicals, revolutionaries and reactionaries in a multi-agent model of class norms, Private communication.

[3] P. W. Anderson, More is different, *Science*, 1972, **177**: 393–396.

[4] E. Bonabeau, M. Dorigo, and G. Theraulaz, *Swarm Intelligence: From Natural to Artificial Systems*, Oxford University Press, New York, 1999.

[5] J. Han, L. Guo, and M. Li, Guiding a group of locally interacting autonomous mobile agents, in *The 2nd Chinese-Swedish Conference on Control*, Beijing, October 16–17, 2004.

[6] C. Reynolds, Flocks, birds, and schools: a distributed behavioral model, *Computer Graphics*, 1987, **21**: 25–34. The demo can be downloaded from http://www.red3d.com/cwr/boids/applet/.

[7] T. Vicsek, A. Czirok, and E. B. Jacob, I. Cohen, and O. Schochet, Novel type of phase transitions in a system of self-driven particles, *Physical Review Letters*, 1995, **75**: 1226–1229.

[8] J. Desai, J. Otrowski, and V. Kumar, Modeling and control of formations of nonholonomic robots, *IEEE Trans. on Robotics and Automation*, 2001, **17**(6): 905–908.

[9] X. F. Wang and G. Chen, Pinning control of scale-free dynamical networks, *Physica*, 2002, **310**(A): 521–531.

[10] A. Jadbabaie, J. Lin, and A. S. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules, *IEEE Trans. on Automatic Control*, 2003, **48**: 988–1001.

[11] N. E. Leonard and E. Fiorelli, Virtual leaders, artificial potentials and coordinated control of groups, in *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, FL, December 2001, 2968–2973.

[12] Project of soft control. http://complex.amss.ac.cn/hanjing/softcontrol/. Videos and demos can be downloaded from http://complex.amss.ac.cn/hanjing/softcontrol/demo.html.

[13] Synthetic Reality Project at Carnegie Mellon. http://www-2.cs.cmu.edu/∼claytronics/.

[14] W. S-Y. Wang, J. Ke, and J. W. Minett, Computational studies of language evolution, in *Computational Linguistics and Beyond* (ed. by R. Huang and W. Lenders), Nankang, Jiangxi, 2004, 65–106.