Feedback and Uncertainty: Some Basic Problems and Results☆

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Abstract

This paper will review some fundamental results in the understanding of several basic problems concerning feedback and uncertainty. First, we will consider adaptive control of linear stochastic systems, in particular, the global stability and optimality of the well-known self-tuning regulators, designed by combining the least-squares estimator with the minimum variance controller. This natural and seemingly simple case had actually been a longstanding central problem in the area of adaptive control, and its solution offers valuable insights necessary for understanding more complex problems. Next, we will discuss the theoretical foundation of the classical proportional-integral-derivative (PID) control, to understand the rationale behind its widespread successful applications in control practice where almost all of the systems to be controlled are nonlinear with uncertainties, by presenting some theorems on the global stability and asymptotic optimality of the closed-loop systems, and by providing a concrete design method for the PID parameters. Finally, we will consider more fundamental problems on the maximum capability and limitations of the feedback mechanism in dealing with uncertain nonlinear systems, where the feedback mechanism is defined as the class of all possible feedback laws. Some extensions and perspectives will also be discussed in the paper.

Keywords: Feedback, uncertainty, nonlinear systems, adaptive control, least-squares, PID control, stability.

☆This paper was supported by the National Natural Science Foundation of China under grant No.11688101, and is based on the plenary (Bode) lecture delivered by the author at 58th IEEE Conference on Decision and Control held at Nice, France, December 11-13,2019.

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1. Introduction

As is well-known, feedback is ubiquitous and is the most basic concept of automatic control. It is the systematic and quantitative investigation of the feedback mechanism that distinguishes the area of automatic control from all other branches of science and technology. In fact, feedback control has been a central theme in control systems, and tremendous progress has been made in both theory and applications(see,e.g.,[1],[2]). One celebrated example is the Bode integral formula[3] on sensitivity functions, which reveals a fundamental limitation of feedback, and has had a lasting impact on the field[4]. Uncertainty is ubiquitous too, either internal or external. It is the existence of uncertainty that necessitates the use of feedback in control systems. Mathematically, uncertainty is usually described by a set, either parametric or functional.

The feedback control of uncertain dynamical systems is by definition the control of all possible systems relating to the uncertainty set, by using available system information. It is worth to mention that modeling, identification and feed-forward are also instrumental for controller design, but we will focus on feedback and uncertainty in this paper to understanding their quantitative relationship, by presenting a series of basic theorems.

To be specific, we will in this paper provide a review of some fundamental results on the following three classes of control problems: 1) The self-tuning regulator(STR), which is a nonlinear feedback law for adaptive control of linear uncertain stochastic systems; 2) The classical PID control, which is a linear feedback law consisting of three terms (proportional-integral-derivative, PID) about the control error, but applied to nonlinear uncertain dynamical systems; 3) The maximum capability of feedback, which concerns with nonlinear feedbacks for nonlinear uncertain systems. The main reasons for a review of the above three classes of control problems will be delineated one by one below.

The reasons for a review of STR thoery. Firstly, the STR is a most basic and natural adaptive controller, see Figure 1 for the block diagram of such adaptive systems. It is basic since it deals with the control of linear plants with unknown parameters and with random noises, and it is natural because it is constructed by combining the online least-squares(LS) estimation with the minimum vari-

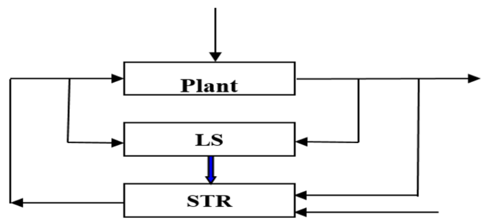

Figure 1: LS-based STR

ance(tracking) control. Thus, one may naturally wonder whether or not such a combination of "optimal estimator" with "optimal controller" will give rise to an asymptotically optimal adaptive controller. It is worth noting that if one ignores the existence of noises, then the control problem may become trivial in theory, because in that case the unknown parameters could be solved exactly within finite steps under suitably designed input signals.

Secondly, the increasing influences and prevalent applications of machine learning algorithms are usually performed not in closed-loops, and it would be natural to further consider the combination of machine learning algorithms with online decision making, and for that the understanding and investigation of the basic STR appear to be helpful.

Thirdly, the STR has played an important historical role in adaptive control. As pointed out by Lennart Ljung in his preamble[5] to the reprint of the seminal paper[6] published by Åström and Wittenmark in 1973, "The paper by Åström and Wittenmark had an immediate impact. Literally thousands of papers on self-tuning regulation, both theoretical and applied, appeared in the next decade. On the theoretical front, the paper left open the question of convergence and stability and this inspired much subsequent research. ......The lasting influence of the paper is perhaps best judged by the fact that today there are many thousands of control loops in practical use that have been designed using the self-tuning concept......The self-tuning regulator revitalized the field of adaptive control that had lost, in the early 1970s, some of its earlier lustre".

Lastly, the research process leading to the final resolution of the longstanding problem on global stability and asymptotic optimally of the STR is of rich enlightenment. Besides the landmark contribution of Aström and Wittenmark[6] as just mentioned, we remark that Kalman [7] used the self-tuning idea by combining the least-squares parameter estimation with the deadbeat control for linear systems without noises. Due to the impetus for the need to analyse the STR, extensive research effort bas been devoted to the convergence analysis of L-S for linear regression models with stochastic regressors, under certain excitation conditions imposed on the system signals(see, e.g.,[8, 9, 10, 11, 12, 13, 14, 15]). A widely influential work was made by Goodwin, Ramadge and Caines[16, 17] who had established the global stability and optimality of the closed-loop systems, under an adaptive controller designed by replacing the LS estimation with a stochastic approximation(SA)algorithm in the STR. The convergence of the SA algorithm in adaptive control systems was later investigated in [18] and [19]. However, because the SA-based adaptive controller has a much slower convergence rate than the LS-based STR, the theoretical investigation of STR had continued to attract research attentions(see, e.g.,[14, 20]). It was not until the publication of the paper by Guo and Chen [21] that a fairly complete solution to the global stability and optimality of the STR was found, and it was later shown that the STR does indeed has the best possible rate of convergence [22].

The reasons for a review of PID theory. It is well-known that the classical PID(proportional-integral-derivative) control is a linear combination of three terms consisting of the "present-past-future" output errors, see Figure 2. This

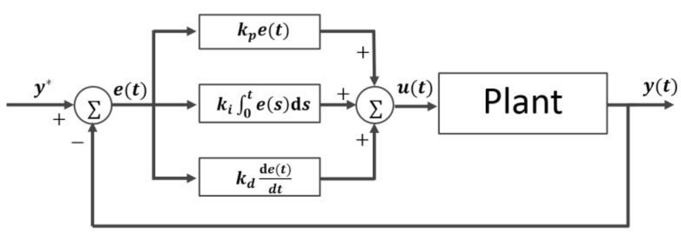

Figure 2: PID control

simple controller is model-free, data-driven and easy-to-use, but the three PID parameters are case dependent, which are usually tuned by experiences or experiments or by both. As is well-known, the PID controller can eliminate steady state offsets via the integral action, and can anticipate the tendency through the derivative action. Also, the Newton's second law corresponds to a second order differential equation, which is just suitable for the PID control.

Despite the remarkable progresses in modern control theory, the classical PID control is still the most widely used one in engineering systems, and as pointed out in [23], "we still have nothing that compares with PID". For example, more than 95% control loops are of PID type in process control, and the PID controller can be said as the "bread and butter" of control engineering[24]. There are also various PID software packages, commercial PID hardware modules, and patented PID tuning rules [25]. Thus, it is not exaggerating to say that there would be no modern civilization if there were no PID control.

The PID actually has a long history, see, e.g.[24, 26] and the references therein. The proportional(P) feedback was used in a centrifugal governor for regulating the speed of windmills in the mid of eighteenth century, and was later used in a similar way for steam engines by James Watt in 1788. The complete form of PID consisting of the three terms was emerged at least one hundred years ago(see,e.g.,[26],[27]), and a well-known tuning rule for the three parameters of PID was proposed by Ziegler and Nichols[28] based on experiments conducted in either time domain or frequency domain. Due to the various advantages of PID as mentioned above, the PID has received continued research attention until recently, but most are on linear systems (see, e.g., [24, 29, 30, 31, 32, 33, 34]), except a few related papers on nonlinear systems(see,e.g., [35, 36, 37, 38, 39]).

The main reasons that we are interested in the theoretical foundation of PID control are as follows: Firstly, almost all practical systems are nonlinear with uncertainties, but almost all theoretical studies focus on linear systems and the tuning of the PID parameters is case dependent. Clearly, there is lack of a satisfactory theory for the PID control and the gap between theory and practice of PID needs to be filled up by control theorists. Secondly, to justify the remarkable practical effectiveness of the PID controllers, we need to face nonlinear uncertain dynamical systems, and to understand the rationale and capability of the PID controller. Thirdly, a large number of practical PID loops are believed to be poorly tuned[25], and better understanding of the PID control may improve its widespread practice and so contribute to better product quality[24].

In 2017, Zhao and Guo[40] made a notable step in the theoretical investigation of PID control for second order nonlinear uncertain systems. They proved that the closed-loop systems controlled by the PID will be globally stable and asymptotically optimal, as long as the three PID parameters are chosen arbitrarily from a three dimensional (open and unbounded) parameter set, which can be constructed explicitly by using the upper bounds of the partial derivatives of the nonlinear functions involved. They also discussed some cases where the choice of the PID parameters are necessary for global stabilization. The results in [40] were later extended to high-dimensional uncertain nonlinear state space models in [41], and the design of the three PID parameters were further refined by providing a concrete formula with guaranteed nice transient control performances[42]. Details will be discussed in Section 3.

The reasons for a review of the capability of feedback. Both the STR and the PID mentioned above are special forms of feedback laws. By feedback capability we mean the maximum capability and fundamental limitations of the feedback mechanism in dealing with uncertainties, which is defined as the class of all possible feedback laws (not restricted to a special class of feedbacks). Our emphasis will be placed on the quantitative relationship between feedback and uncertainty, to understand how much uncertainty in the class of unknown functions (denoted by $\mathcal{F}$) can be dealt with by the feedback mechanism(denoted by U). Such control problems may be roughly depicted by the block diagram below(Figure 3) and will be discussed rigorously in Section 4.

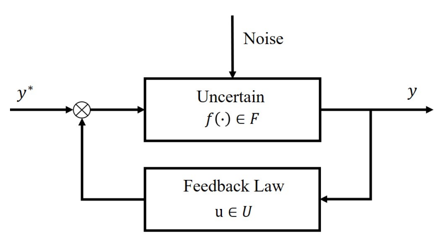

Figure 3: Feedback and uncertainty

We remark that we are interested to know not only what the feedback mechanism can do, but also what the feedback mechanism cannot do, in the presence of large structural uncertainties. We would like to mention that this study is not merely of intellectual curiosity. In fact, the understanding of the maximum capability of feedback can encourage us in improving the controller design to reach or approach the maximum capability, and may help us in alleviating the workload of modeling and identification. Moreover, the investigation of the fundamental limitations of feedback may prevent us from wasting time and energy on searching for a feedback controller that does not exist, and may also alert us of the danger of being unable to control uncertain systems when the size of the uncertainty reaches the limit established.

Given the remarkable progresses in modern control theory made over the past 60 years, it can be said that the most relevant research areas on feedback and uncertainty are adaptive control and robust control, among others. However, due to the fundamental differences with our problem formulations, only a few existing results address the maximum capability and fundamental limitations of the feedback mechanism, see, e.g.,[43] for a class of linear stochastic systems with a white noise control channel, and [44] for a class of uncertain linear systems. In Section 4, we will present a series of fundamental theorems concerning the maximum capability and fundamental limitations of the feedback mechanism for several basic classes of uncertain nonlinear dynamic systems. This line of work was initiated in [45] and has been partly summarized in the plenary lecture [46].

In the next three sections, we will briefly present some concrete and basic results on STR, PID and feedback capability respectively, mainly for minimum-phase uncertain dynamical control systems.

2. Theory of Self-Tuning Regulators

Consider the following discrete-time uncertain linear control systems with additive noises,

$$y_{k+1} + a_1 y_k + \cdots + a_p y_{k-p+1} = b_1 u_k + \cdots + b_q u_{k-q+1} + w_{k+1},\ \ k \geq 0,$$

where u_k, y_k and w_k are the scalar system input, output and random noises, respectively. The coefficients a_i and b_j are assumed to be unknown, and p and q are assumed to be known upper bounds for the true orders of the systems.

The above systems can be rewritten into the following standard linear regression form:

$$y_{k+1} = \boldsymbol{\theta}^{\tau} \boldsymbol{\varphi}_k + w_{k+1}, \quad k \geq 0,$$

where the regression vector and parameter vector are defined respectively by

$$\begin{aligned} \boldsymbol{\varphi}_k &= [y_k, \ldots, y_{k-p+1}, u_k, \ldots, u_{k-q+1}]^{\tau}, \\ \boldsymbol{\theta} &= [-a_1, \ldots, -a_p, b_1, \ldots, b_q]^{\tau}. \end{aligned}$$

To established a theory for the control of this class of uncertain systems, we need to introduce the following standard assumptions:

A1. The noise $\{w_k, \mathcal{F}_k\}$ is a martingale difference sequence, and there exists a constant $\beta > 2$ such that the β-th order conditional moments satisfy

$$\sup_{k \geq 0} \mathbb{E}[|w_{k+1}|^{\beta} | \mathcal{F}_k] < \infty, \quad a.s.$$

where $\{\mathcal{F}_k\}$ is a family of non-decreasing σ-algebras.

A2. The system is of minimum phase.

A3. The reference signal $\{y_k^*\}$ is a bounded deterministic sequence.

Note that if $\{u_k\}$ is an output feedback sequence, then $\{y_i, 0 \leq i \leq k\}$ will be measurable to $\mathcal{F}_k$.

Let us consider the tracking control problem where the purpose is to minimize the following one-step ahead tracking performance:

$$J_k = \min_{u_k \in \mathcal{F}_k} E(y_{k+1} - y_{k+1}^*)^2, \;\; k \geq 1.$$

By Assumption A1, it is easy to see that at any time k, the best prediction to y_{k+1} is the conditional mathematical expectation, i.e., $\mathbb{E}[y_{k+1}|\mathcal{F}_k] = \boldsymbol{\varphi}_k^T \boldsymbol{\theta}$. Therefore, if the parameter vector $\boldsymbol{\theta}$ were knew, then the optimal control could be solved by setting

$$\mathbb{E}[y_{k+1}|\mathcal{F}_k] = y_{k+1}^*$$

to get an explicit expression

$$u_k = \frac{1}{b_1}(a_1 y_k + \cdots + a_p y_{k-p+1} - b_2 u_{k-1} - \cdots - b_q u_{n-q+1} + y_{k+1}^*)$$

with the following optimal cost:

$$J_k = \mathbb{E}[w_{k+1}^2 | \mathcal{F}_k], \ \ \forall k \geq 1.$$

In the current case where the parameter vector $\boldsymbol{\theta}$ is unknown, we use the following well-known least-squares(LS) method to estimate it:

$$\boldsymbol{\theta}_k = \arg\min_{\boldsymbol{\theta} \in \mathbb{R}^{p+q}} \sum_{j=1}^{k} (y_j - \boldsymbol{\varphi}_{j-1}^{\tau} \boldsymbol{\theta})^2, \ \ \forall k \geq 1,$$

which can be solved explicitly as

$$\boldsymbol{\theta}_k = \left(\sum_{j=1}^{k} \boldsymbol{\varphi}_{j-1} \boldsymbol{\varphi}_{j-1}^{\tau} \right)^{-1} \left(\sum_{j=1}^{k} \boldsymbol{\varphi}_{j-1} y_j \right),$$

and calculated recursively by

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + a_k P_k \boldsymbol{\varphi}_k (y_{k+1} - \boldsymbol{\varphi}_k^{\tau} \boldsymbol{\theta}_k),$$

$$P_{k+1} = P_k - a_k P_k \boldsymbol{\varphi}_k \boldsymbol{\varphi}_k^{\tau} P_k, \quad a_k = (1 + \boldsymbol{\varphi}_k^{\tau} P_k \boldsymbol{\varphi}_k)^{-1},$$

where the initial estimate $\boldsymbol{\theta}_0 \in \mathbb{R}^{p+q}$, and the initial positive definite matrix $P_0 \in \mathbb{R}^{(p+q)\times(p+q)}$ can be chosen arbitrarily.

By using the above online LS estimate, one can construct an adaptive predictor $\widehat{y}_{k+1}$ based on the "certainty equivalence principle", i.e.,

$$\widehat{y}_{k+1} = \boldsymbol{\varphi}_k^{\tau} \boldsymbol{\theta}_k.$$

Now, let $\widehat{y}_{k+1} = y_{k+1}^*$, the STR can be expressed as follows:

$$u_k = \frac{1}{b_{1k}} (a_{1k} y_k + \cdots + a_{pk} y_{k-p+1} - b_{2k} u_{k-1} - \cdots - b_{qk} u_{n-q+1} + y_{k+1}^*),$$

where a_{ik}, b_{ik} are the corresponding components of the LS estimate $\boldsymbol{\theta}_k$.

To avoid possible zero divisor problems in the controller expression above, b_{1k} can be modified slightly and replaced by, e.g.,

$$\hat{b}_{1k} = \begin{cases} b_{1k}, & \text{if } |b_{1k}| \geq \frac{1}{\sqrt{\log r_{k-1}}} \\ b_{1k} + \frac{sign(b_{1k})}{\sqrt{\log r_{k-1}}}, & \text{otherwise} \end{cases}$$

where $sign(\cdot)$ is the sign function, and r_k is defined by

$$r_k = e + \sum_{i=0}^{k} \|\varphi_i\|^2, \quad k \geq 1.$$

For the above defined STR that combines the least-squares estimator with the minimum variance (tracking) controller, we are interested to know whether or not the closed-loop control system performs well. To be specific, we are interested in answering the following three basic questions: 1) Is the closed-loop adaptive system globally stable? 2) Is the system tracking error asymptotically optimal? 3) Does the STR enjoy the best possible rate of convergence? As mentioned in the introduction, these basic theoretical issues have been long-standing open problems in adaptive control theory. One may curiously to ask why the analysis of such a naturally defined STR is so complicated? The basic reason is that the closed-loop systems are characterized by a set of complicated nonlinear stochastic dynamical equations, where the closed-loop system signals are nonstationary and strongly correlated, and there is no useful statistical properties available a priori. Since the LS is a key ingredient of STR, one may wonder whether or not the extensively studied convergence theory on LS will be helpful. Unfortunately, the verification of even the weakest possible convergence condition for LS [12] is still quite hard, since it requires that the stability of the closed-loop systems be established by other methods. In fact, how to get out of possible "circular arguments" between system stability and estimate convergence is a central issue in adaptive theory.

To sidestep such "circular arguments" in the analysis, we consider the notion of regret of tracking. Note that the performance of adaptive tracking depends essentially on the quality of the adaptive predictor. The difference between the best prediction and the adaptive prediction (or tracking signal) may be referred to as the "regret" denoted by

$$R_k = (\mathbb{E}[y_{k+1}|\mathcal{F}_k] - \widehat{y}_{k+1})^2,$$

which is usually not zero due to the existence of the unpredictable noises.

However, one may evaluate the "averaged regret" defined by

$$\frac{1}{n}\sum_{k=1}^{n} R_k.$$

By using Assumption A1, it is not difficult to show that the global stability and optimality will follow once the above averaged regret tends to zero as n increases to infinity[47]. By introducing a new method for analysing the nonlinear closed-loop dynamics, Guo and Chen in[21] was able to establish the global stability and asymptotic optimality of the STR, which is presented in the following theorem:

Theorem 2.1. Under Assumptions A1-A3, the averaged regret tends to zero. In other words, the closed-loop control system of STR is globally stable, i.e., for any initial condition y_0,

$$\limsup_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}(y_k^2 + u_k^2) < \infty, \qquad a.s.,$$

and asymptotically optimal, i.e.,

$$\limsup_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}(y_k - y_k^*)^2 = min, \qquad a.s..$$

where "min" denotes the minimum tracking error which equals to the upper limit of the averaged conditional variances of the noises.

To demonstrate that the STR does indeed enjoy the best possible rate of convergence, we present the following logarithm law for the accumulated regret of tracking[22]:

Theorem 2.2. Under some additional assumptions, the closed-loop control system will have the following logarithm law for the accumulated regret of tracking:

$$\lim_{n\to\infty} \frac{1}{\log n}\sum_{k=1}^{n}(\mathbb{E}[y_{k+1}|\mathcal{F}_k] - y_{k+1}^*)^2 = dim(\theta)\sigma_w^2, \qquad a.s.,$$

where $dim(\theta)$ is the dimension of the unknown parameter vector, σ_w^2 is the conditional variance of the noises(assumed to be constant for simplicity).

Remark 2.1. (i) The details of Theorem 2.2 is given in [22] where the additional assumptions can be made either on the high-frequency gain b_1 or on the reference signal y_k^*. Also, a discussion why $O(\log n)$ is the minimal order of magnitude that one may at most expect to achieve for the accumulated regret is found in [48]. (ii) In the analysis of STR, the convergence of the averaged regret of adaptive prediction is more relevant than the convergence of the LS itself. A detailed analysis of LS can provide a sharp bound for a certain accumulated weighted regret regardless of the input signal, which turns out to be critical for further analysis of the nonlinear closed-loop stochastic systems under STR. (iii) The martingale theory has played a fundamental role in dealing with non-stationary and correlated signals or data. This may continue to be so when we deal with more complicated data or signals that are generated from complex stochastic feedback systems, where independency and stationarity properties are not available.

Remark 2.2. Concerning about other related problems and results in stochastic adaptive control, we briefly mention the following facts: (i) Theorem 2.1 can be extended to multi-input and multi-output(MIMO) minimum phase linear stochastic systems with colored noises[21], to linear stochastic systems with multiple delay and model reference[49], and to a class of linearly parameterized nonlinear stochastic systems[50]. (ii) For adaptive control of non-minimum phase linear stochastic systems, a bottleneck problem is how to guarantee the controllability of the estimated model without resorting to projection to a known convex controllability domain, which can be resolved (see [51]) by a random regularization approach combined with the self-convergence property established in [51]for the weighted LS proposed in [52]. Based on this, an asymptotically optimal adaptive LQG control is given in[53]. (iii) For adaptive control of linear time-varying stochastic systems in discrete-time with unknown Markovian jump parameters, a necessary and sufficient condition is given in[54] for global adaptive stabilization. This condition is different from that known for the non-adaptive case[55], and reveals an attenuation of feedback capability in the presence of jump parameter uncertainty. There are also many investigations in the continuous-time(see, e.g.,[56] and the references therein), and a complete characterization is yet to be found.

3. Theory and Design of PID Control

Let $x(t)$, $v(t)$ and $a(t)$ be the position, velocity and acceleration of a moving body in $\mathbb{R}$ with mass m at time instant t. Assume that the external forces acting on the body consist of f and u, where $f = f(x,v)$ is a nonlinear function of both the position x and velocity v, where u is the control force. Then by the Newton's second law we know that

$$ma(t) = f(x(t),v(t)) + u(t).$$

Let us denote $x_1(t) = x(t)$ and $x_2(t) = \frac{dx(t)}{dt} = \dot{x}(t)$, and without loss of generality, we assume that the body has unit mass. Then the state space equation of the above basic mechanic system under PID control is

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(x_1, x_2) + u(t), \\ u(t) = k_p e(t) + k_i \int_0^t e(s)ds + k_d \frac{de(t)}{dt}, \end{cases} \tag{1}$$

where $x_1(0)$, $x_2(0) \in \mathbb{R}$ and $e(t) = y^* - x_1(t)$, and $f(x_1,x_2)$ is an uncertain nonlinear function.

Next, let us introduce the class of uncertain functions defined by

$$\mathcal{F}_{L_1,L_2} = \left\{ f \in C^1(\mathbb{R}^2) \middle| \frac{\partial f}{\partial x_1} \leq L_1, |\frac{\partial f}{\partial x_2}| \leq L_2, \forall x_1, x_2 \in \mathbb{R} \right\}$$

where L_1 and L_2 are positive constants, and $C^1(\mathbb{R}^2)$ denotes the space of all functions from $\mathbb{R}^2$ to $\mathbb{R}$ which are locally Lipschitz in (x_1,x_2) with continuous partial derivatives.

It is quite obvious that the "size" of the uncertainty class $\mathcal{F}_{L_1,L_2}$ will increase whenever L_1 or L_2 increases. We remark that L_1 and L_2 correspond to the upper bounds of the "anti-stiffness" and the "anti-damping" coefficients of the nonlinear systems, respectively.

Given the upper bounds L_1 or L_2, we can construct the following 3-dimensional parameter set from which the three PID parameters can be chosen arbitrarily:

$$\Omega_{pid} = \left\{ \begin{bmatrix} k_p \\ k_i \\ k_d \end{bmatrix} \middle| k_p > L_1,\, k_i > 0,\, (k_p - L_1)(k_d - L_2) > k_i + L_2\sqrt{k_i(k_d + L_2)} \right\}.$$

It is easy to see that this set is open and unbounded.

We are now in a position to present the first main result concerning PID control.

Theorem 3.1. Consider the above PID controlled nonlinear uncertain system(1). Then, whenever $(k_p,k_i,k_d) \in \Omega_{pid}$, the closed-loop control system will satisfy

$$\lim_{t\to\infty} x_1(t) = y^*, \qquad \lim_{t\to\infty} x_2(t) = 0,$$

exponentially fast, for any $f \in \mathcal{F}_{L_1,L_2}$, any initial state $(x_1(0), x_2(0)) \in R^2$, and any setpoint $y^* \in \mathbb{R}$.

We remark that the above theorem was first proven in[40], which can be extended to high-dimensional systems [41], where time-varying uncertain nonlinear functions have also been considered.

Remark 3.1. Firstly, Theorem 3.1 is a global result since it depends on neither the initial state nor the setpoint. Secondly, whatever methods one may use in choosing the PID parameters, the closed-loop systems will have the desired properties as established in Theorem 3.1 as long as the three parameters belong to Ω_{pid}. Thirdly, Theorem 3.1 demonstrates that the PID control has large-scale and two-sided robustness in the following sense: On the system structure side, for any given PID controller with $(k_p,k_i,k_d) \in \Omega_{pid}$, it can deal with the whole class of nonlinear uncertain systems described by $f \in \mathcal{F}_{L_1,L_2}$; while on the controller parameter side, any parameter triple (k_p,k_i,k_d) in the unbounded open set Ω_{pid} can give rise to a globally stabilizing PID controller with exponentially vanishing output errors. This remarkable property partly explains the wide applicability of the PID control. Finally, we mention that, since the selection of the PID parameters has much flexibility, more performance requirements including the transient may be further studied by optimizing the PID parameters from the set Ω_{pid}. Since Theorem 3.1 only gives a sufficient condition for the choice of the PID parameters, a natural question is: is Ω_{pid} a necessary parameter set? To answer this basic question, we need some additional constrains on the class of uncertain functions.

We first consider an affine situation and introduce the following function class:

$$\mathcal{G}_{L_1,L_2} = \left\{ f \in C^2(\mathbb{R}^2) \middle| \frac{\partial f}{\partial x_1} \leq L_1,\ \frac{\partial f}{\partial x_2} \leq L_2,\ \frac{\partial^2 f}{\partial x_2^2} = 0,\ \forall x_1,\ x_2 \in \mathbb{R} \right\},$$

where $L_1 > 0$, $L_2 > 0$ are constants and $C^2(\mathbb{R}^2)$ is the space of twice continuously differentiable functions from $\mathbb{R}^2$ to $\mathbb{R}$. The following theorem was established in [40]:

Theorem 3.2. For any $f \in \mathcal{G}_{L_1,L_2}$, any initial conditions, and any setpoint $y^* \in \mathbb{R}$, the control system satisfies

$$\lim_{t\to\infty} x_1(t) = y^*, \qquad \lim_{t\to\infty} x_2(t) = 0,$$

if and only if the PID parameters (k_p,k_i,k_d) belong to the following 3-dimensional set:

$$\Omega'_{pid} = \left\{ \begin{bmatrix} k_p \\ k_i \\ k_d \end{bmatrix} \middle| k_p > L_1,\ k_i > 0,\ (k_p - L_1)(k_d - L_2) > k_i \right\}$$

Remark 3.2. By using Theorem 3.2, we may investigation the capability of PID in the following sense: Given a PID controller with parameter (k_p,k_i,k_d)($k_i > 0$), what is the largest possible class of nonlinear uncertain functions it can deal with? Note that the "size" of $\mathcal{G}_{L_1,L_2}$ can be "measured" by (L_1, L_2). Hence by Theorem 3.2, the boundary of the following set

$$\left\{ (L_1,L_2) \in \mathbb{R}^2 |\ L_1 < k_p,\ L_2 < k_d - k_i(k_p - L_1)^{-1} \right\}$$

may reflect the maximum capability of this PID controller in dealing with uncertain systems described by $\mathcal{G}_{L_1,L_2}$.

Next, we consider the case where $(y^*, 0)$ is an equilibrium point of the open-loop systems, i.e. $f(y^*, 0) = 0$. In this case, the I-term is not necessary for regulation. Define the following function class:

$$\mathcal{F}_{L_1,L_2,y^*} = \left\{ f \in C^1(\mathbb{R}^2) \middle| \frac{\partial f}{\partial x_1} \leq L_1,\, \frac{\partial f}{\partial x_2} \leq L_2,\, \forall x_1,\, x_2,\, f(y^*,0) = 0. \right\}$$

We have the following result which again gives a necessary and sufficient condition for the design of the controller parameters.

Theorem 3.3. Consider the following nonlinear uncertain systems under PD control:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(x_1, x_2) + u(t), \\ u(t) = k_p e(t) + k_d \frac{de(t)}{dt}. \end{cases} \tag{2}$$

Then for any $f \in \mathcal{F}_{L_1,L_2,y^*}$, we have

$$\lim_{t\to\infty} x_1(t) = y^*, \qquad \lim_{t\to\infty} x_2(t) = 0,$$

if and only if the PD parameters (k_p,k_d) lie in the following two dimensional set:

$$\Omega_{pd} = \left\{ (k_p,\, k_d) \middle| k_p > L_1,\, k_d > L_2 \right\}.$$

The proof is provided in [40] which follows from the Markus-Yamabe theorem. This theorem was originally a conjecture(also called Jacobian conjecture) on global asymptotic stability of ordinary differential equations proposed by Markus and Yamabe in 1960 [57], and after several decades this conjecture had been proven to be true for nonlinear systems in the plane[58, 59].

Remark 3.3. (A further formula for PID parameters). To take the transient performances of PID control into account, one may further specify the PID parameters from the parameter set Ω_{pid} given in Theorems 3.1. One way to do this is recently provided by the following formula[42]:

$$\begin{cases} k_p = k_{ap} + \omega_0 k_{ad}, \\ k_i = \omega_0 k_{ap}, \\ k_d = k_{ad} + \omega_0. \end{cases}$$

where ω_0 can be taken as any positive constant above a lower bound that can be derived from the structure of Ω_{pid} [42], and where (k_{ap}, k_{ad}) is a given pair of real numbers such that the following second order polynomial has zeros in the left-half plane:

$$s^2 + k_{ad}s + k_{ap} = 0.$$

Thus, the closed-loop error equation is expected to have "poles" determined by (k_{ap},k_{ad}), since the PID controller can be reparameterized as :

$$u(t) = k_{ap}e(t) + k_{ad}\dot{e}(t) - \hat{f}_t + \ddot{r}(t),$$

where $e(t) = r(t) - x_1(t)$, $r(t)$ is a designed process with prescribed transient behaviours and with steady state value y^* (see [42]), and where $\hat{f}_t$ is defined by

$$\hat{f}_t = -\omega_0 \left\{ k_{ad}e(t) + k_{ap} \int_0^t e(s)ds + \dot{e}(t) \right\},$$

which can serve as a nice online estimator for the uncertain dynamics f. Moreover, the larger the constant ω_0, the better the performances of estimation and control(see [42] for details).

We would like to mention that the above formula stems from the inherent connection between the PID and the ADRC (Active disturbance rejection control). The ADRC was proposed by J.Q. Han in [60](see also[61]) and has been successfully applied to various industrial processes. A key ingredient of ADRC is an extended state observer (ESO) used for estimating the uncertain nonlinear dynamics. The ESO may be designed as a linear one[62], and the reduced order linear ESO[63] will give the above estimator $\hat{f}_t$ for the unknown nonlinear dynamics f, see[42] for details.

Remark 3.4. Related problems on MIMO, stochastic, and multi-agent nonlinear uncertain systems may also be investigated. Extensions to nonlinear systems with relative degree greater than two can be carried out by using the extended PID controller[64]. Semi-global results may be obtained if the partial derivatives of the uncertain functions are not bounded but some upper bound functions are known[64]. It would be interesting to further consider situations such as (i) saturation, dead-zone, time-delay, sampled data, and observation noises; (ii) extensions of the classical PID to, e.g., adaptive and nonlinear PID, for more general uncertain nonlinear systems; and (iii) making more efforts in combing classical control ideas with modern mathematical methods.

4. Maximum Capability of Feedback Mechanism

To investigate the maximum capability and fundamental limitations of the feedback mechanism, we need to give a precise definition of feedback first.

Feedback needs information, and information can be classified as *prior* information and *posterior* information. The *prior* information refers to the knowledge about the plant before exerting the control force, and the *posterior* information means the knowledge about the plant obtained via the running of the control systems. The *posterior* information is usually contained in the measured input-output data of the systems, denoted by $\{y_0, u_0; y_1, u_1; \cdots, y_t\}$ at any time instant t, where u_t and y_t are the system input and output signals belonging to $\mathbb{R}^p$ and $\mathbb{R}^m$, respectively . If the input is a feedback signal, then the *posterior* information can simply be denoted as $\{y_0, y_1, \cdots, y_t\}$. It is the *posterior* information that makes it possible for feedback to reduce the influence of uncertainties on the control systems.

By a feedback signal u_t we mean that there is a measurable mapping

$$h_t : \mathbb{R}^{m(t+1)} \to \mathbb{R}^p$$

such that

$$u_t = h_t(y_0, y_1, \cdots, y_t)$$

A feedback law u is defined as

$$u = \{u_t, \quad t \geq 0\},$$

and the feedback mechanism is defined as U:

$$U = \{u \,|\, u \text{ is any feedback law}\}.$$

We are interested in how much uncertainty can the feedback mechanism U deal with in control systems. Since stabilization is a primary objective for any control systems, we can then define the capability of the feedback mechanism as the capability in globally stabilizing uncertain dynamical systems, measured by the largest possible class $\mathcal{F}$ of uncertainties that can be dealt with by the whole feedback mechanism U, see also Figure 3 for a block diagram of the problem formulation. Note that the observed output y_t of a control system depends on both the uncertain function $f \in \mathcal{F}$ and the control law $u \in U$, we may write out this dependence explicitly as $y_t(f, u)$. Mathematically, the maximum capability of the feedback mechanism can be defined as

$$\sup_{\mathcal{F}} \left\{ \text{size}\mathcal{F} : \inf_{u \in U} \sup_{f \in \mathcal{F}} \sup_{t \geq 0} |y_t(f, u)| < \infty, \ \forall\, y_0 \in \mathbb{R}^m \right\}.$$

Of course, one may immediately realize that it is not easy to get a complete solution in general. Before pursuing further, we state a simple fact as follows.

A Basic Fact. Let $\mathcal{F}_0$ and $\mathcal{F}$ be two classes of functions satisfying

$$\mathcal{F}_0 \subset \mathcal{F}.$$

If the uncertain system corresponding to the function class $\mathcal{F}_0$ cannot be stabilized by the feedback mechanism, then neither for systems corresponding to the larger function class $\mathcal{F}$.

This fact implies that once we have established an impossibility theorem on feedback capability for a class of uncertain systems, this theorem will continue to be true for any larger class of uncertain systems. Throughout this section, we will consider single-input and single-output(SISO) uncertain systems, and will keep the system models as basic as possible.

In the following, we will consider parametric and nonparametric uncertain systems separately. In both cases, we will first present a theorem for a simple but basic uncertain model class, then present an extended theorem for more general uncertain model classes.

4.1. Parametric Uncertain Systems

Consider the following parametric control systems:

$$y_{t+1} = f(\theta, y_t) + u_t + w_{t+1}.$$

where the unknown parameter $\theta \in \mathbb{R}^1$ lies in a compact set and $\{w_t\}$ is any bounded disturbance sequence. Assume that the sensitivity function satisfies

$$\frac{\partial f(\theta, x)}{\partial \theta} = \Theta(|x|^b), \quad x \to \infty,$$

where $b \geq 0$ is a constant. The notation $\Theta(|x|^b)$ means that there exist two positive constants c_1 and c_2 such that $c_1|x|^b \leq \Theta(|x|^b) \leq c_2|x|^b$ for all sufficiently large x. The following theorem shows that $b = 4$ is a critical value:

Theorem 4.1. The above class of uncertain systems is globally stabilizable by the feedback mechanism **if and only if** $b < 4$.

Remark 4.1. We remark that Theorem 4.1 was first discovered and proven by Guo[45] for the linearly parameterized stochastic case where $f(\theta, y_t) = \theta f(y_t)$, and the present result is given in [65]. We also remark that the noise effect is essential in this theorem, because if there were no noise, one would be able to determine the unknown parameter θ within one step, and consequently, the systems could be stabilized trivially, regardless of the value of $b > 0$. One may naturally ask: why $b = 4$ is a critical value in Theorem 4.1? Roughly speaking, it is determined by the mixed effects of the decreasing rate of the "best" estimation error and the possible growing rate of the sensitivity function. The detailed analysis is quite complicated, but it is closely connected to the following simple fact: The second order polynomial $z^2 - b_1 z + b_1 > 0$, for all $z \in (1, b_1)$, if and only if $b_1 < 4$, see [45].

Next, we consider the following parametric case with multiple unknown parameters:

$$y_{t+1} = \theta^\tau f(y_t) + u_t + w_{t+1} \tag{3}$$

where $\theta \in \Theta \triangleq \{\theta \in \mathbb{R}^p : \|\theta\| \leq R\}$ is a p-dimensional unknown parameter vector, and $\{w_t\}$ is either any bounded disturbance sequence or a Gaussian white noise sequence. Let us denote $f(y_t) \triangleq [f_1(y_t), \cdots, f_p(y_t)]^\tau$ and assume that the function $f(\cdot)$ belongs to:

$$\mathcal{F}(b) = \left\{ f(\cdot) : f_i(x) = \Theta(|x|^{b_i}), \text{ as } x \to \infty \right\},$$

where $b = (b_1 \cdots b_p)$. Without loss of generality, we assume that $b_1 > b_2 > \cdots > b_p > 0$ and $b_1 > 1$. We remark that the case where $b_1 \leq 1$ means that the nonlinear function is bounded by a linear growth rate which can be stabilized globally by an adaptive controller(see, e.g., [50]).

With the exponents b_i defined as above, we introduce a characteristic polynomial as follows:

$$P(z) = z^{p+1} - b_1 z^p + (b_1 - b_2) z^{p-1} + \cdots + (b_{p-1} - b_p) z + b_p.$$

The following result shows that this polynomial can serve as a criterion for stabilizability.

Theorem 4.2. Let $f \in \mathcal{F}(b)$ be a nonlinear function. Then the above uncertain nonlinear dynamical system with $\theta \in \Theta$ is globally stabilizable by the feedback mechanism **if and only if**

$$P(z) > 0, \quad \forall\, z \in (1, b_1).$$

Remark 4.2. When $p = 1$, the above polynomial criterion is equivalent to $b_1 < 4$, the same result as established in Theorem 4.1. The polynomial $P(z)$ was first introduced in [66] with a necessity proof in the stochastic case, the complete proof was given in [67] and [68] for bounded disturbances and white Gaussian noises, respectively. Now, we briefly explain the rationale behind the impossibility or limitations of the feedback mechanism (see, [69] for details). In the case where both the unknown parameter θ and the disturbances $\{w_t\}$ are bounded, one may use a stochastic embedding approach to find the cases where the uncertain systems are not globally stabilizable by the feedback mechanism. One may first express the conditional variance of the output process in terms of the conditional variance of the best prediction error for the uncertain dynamics, then by using the conditional Cramér-Rao-like inequality for dynamical systems to derive a lower bound to the best prediction error for any feedback control, which can be expressed by the Fisher information matrix and the sensitivity function, from which a meticulous analysis of the nonlinear dynamics will finally lead to a connection to the polynomial criterion.

4.2. Nonparametric Uncertain Systems

Let us first consider the following basic nonparametric control system:

$$y_{t+1} = f(y_t) + u_t + w_{t+1}, \quad y_0 \in \mathbb{R}^1 \ ,$$

where $\{w_t\}$ is any bounded sequence of disturbances, and where the unknown function $f(\cdot) \in \mathcal{F} = \{\text{all } \mathbb{R}^1 \to \mathbb{R}^1 \text{ mappings}\}$. We introduce the following Lipschitz norm for a function f in $\mathcal{F}$:

$$\|f\| = \sup_{x \neq y} \frac{|f(x) - f(y)|}{|x - y|} \quad ,$$

which may also be regarded as a kind of sensitivity measure of uncertain functions. Note that a generalized definition for the norm that avoids possible zero divisor problem may also introduced without changing the results to be presented in the following[70]. Now, let us define the following class of functions:

$$\mathcal{F}(L) = \{f \in \mathcal{F} : \ \|f\| \leq L\}.$$

Note that L can serve as a measure of uncertainty: the larger its value, the larger the function class $\mathcal{F}(L)$. The following result is established by Xie and Guo[70].

Theorem 4.3. The above class of uncertain dynamical systems described by $\mathcal{F}(L)$ is globally stabilizable by the feedback mechanism **if and only if**

$$L < \frac{3}{2} + \sqrt{2}.$$

In other words, if $L < \frac{3}{2} + \sqrt{2}$, then there is a feedback law $\{u_t\}$ such that the system is globally stable for any $f \in \mathcal{F}(L)$; and if $L \geq \frac{3}{2} + \sqrt{2}$, then for any feedback law $\{u_t\}$, there is at least one system $f \in \mathcal{F}(L)$, such that the corresponding closed-loop system is not globally stable.

Remark 4.3. One may naturally wonder why $\frac{3}{2} + \sqrt{2}$ is a critical value, since from our problem formulation there is no clue for this. It is not easy to give an intuitive explanation, but instead, we list the following two facts which are closely related to our analysis, where $\frac{3}{2} + \sqrt{2}$ is also critical:

Fact 1: Let $\{y_t\}$ be any sequence satisfying

$$|y_{t+1} - (\text{center})_t| \leq L|y_t - (\text{neighbor})_t|, \quad \forall t \geq 1,$$

where

$$(center)_t = \frac{1}{2}(\min_{0 \leq i \leq t} y_i + \max_{0 \leq i \leq t} y_i), \ (neighbor)_t = y_{i_t}$$

with $i_t = \underset{0 \le i \le t-1}{\text{argmin}} |y_t - y_i|$. Then, any such sequence $\{y_t\}$ is bounded if and only if $L < \frac{3}{2} + \sqrt{2}$.

Fact 2: All solutions of the difference equation $a_{n+1} = L(a_n - a_{n-1}) + \frac{1}{2}a_n$ either converge to zero or oscillate about zero (as illustrated in Figure 4) if and only if $L < \frac{3}{2} + \sqrt{2}$.

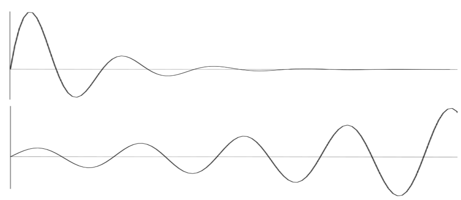

Figure 4: Illustration of the solutions for $L = 0.7$ and 1.1

Next, we consider a generalized uncertainty class consisting of semi-parametric models, where both parametric and nonparametric parts are included. Let $\{g(\theta, \cdot),\ \theta \in \Theta\}$ be a model class with modeling error $f(\cdot) \in \mathcal{F}(L)$ plus a bounded disturbance:

$$y_{t+1} = g(\theta, \phi_t) + f(y_t) + w_{t+1}, \quad t \ge 0,$$

where $\phi_t = [y_t, y_{t-1}, \cdots, y_{t-p+1}, u_t, u_{t-1}, \cdots, u_{t-q+1}]^\tau$.

Assume that $\theta \in \Theta$ where $\Theta \subset \mathcal{R}^m$ is a compact set, that the system is of "minimum phase" in a certain sense, and that the sensitivity function of $g(\cdot, \cdot)$ with respect to the unknown parameter vector θ is bounded by a linear growth rate, etc., see[71] for a complete description of the assumptions. Under these assumptions, the following theorem shows that the additive parametric uncertainties do not change the capability of the feedback mechanism[71].

Theorem 4.4. The above semi-parametric uncertain dynamical systems with $\{(\theta, f) \in (\Theta, \mathcal{F}(L)\}$ are globally stabilizable by the feedback mechanism **if and only if**

$$L < \frac{3}{2} + \sqrt{2}.$$

Remark 4.4. As is well-known, modeling and feedback are two main techniques in dealing with uncertainties. Theorem 4.4 quantitatively shows how modeling and feedback could be complementary in control systems design. In particular, the limitations of feedback may be compensated by improving the quality of modeling, and conversely, the accuracy or demand of modelling may be relaxed by taking the maximum capability of feedback into account.

Before concluding this section, we present the following final remark:

Remark 4.5. In this section, we have presented parts of the basic results on feedback capability obtained over the past 20 years. Further results may be found for both parametric case (e.g.,[72, 65, 73]) and nonparametric case (e.g.,[74, 75, 76]). Fundamental limitations on the sampled-data feedback mechanism with prescribed sampling rate are investigated in [77] followed by a refinement in [78]. We would like to point out that all the impossibility theorems presented in this part enjoy universality in the sense that they are actually valid for any larger class of uncertain systems and for any feedback laws. Also, the main results indicate that the feedback capability depends on both information uncertainty and structural complexity, and that adaptive prediction(estimation) and "sensitivity" functions play a crucial role. Finally, we mention that there appears to be fundamental differences between continuous-time and sampled-data (or discrete-time) feedbacks for uncertain nonlinear systems, when the sampling rate is prescribed.

5. Concluding Remarks

This paper has reviewed some basic problems and results on feedback and uncertainty, focuses on three class of problems, i.e., STR, PID, and feedback capability, which are mainly conducted by the author's research group. Of course, there are many other related problems and results need to be reviewed or mentioned, and there are many more problems remain to be solved or investigated in the future. We would like to make the following perspectives:

(i)The rapid development of information technology makes it possible to investigate more and more complex control systems, and at the same time brings a series of interesting new problems, whose investigation may still depend on our understanding of the basic concepts and problems in the field.

(ii)Mathematical models paly a basic role in control theory even if they may have large uncertainties. However, if the models are not regarded as approximations of the real-world systems and, instead, just taken as an intermediate step for controller design, then great efforts are still needed towards a comprehensive understanding of the boundaries of practical applicability of the controller.

(iii)Furthermore, besides uncertainties, many systems to be controlled or regulated in social, economic, biological, and the future "intelligent" engineering systems, may have their own objectives to pursue. Such complex uncertain systems, may not belong to the traditional framework of control or game theory, and call for more research attention[79].

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